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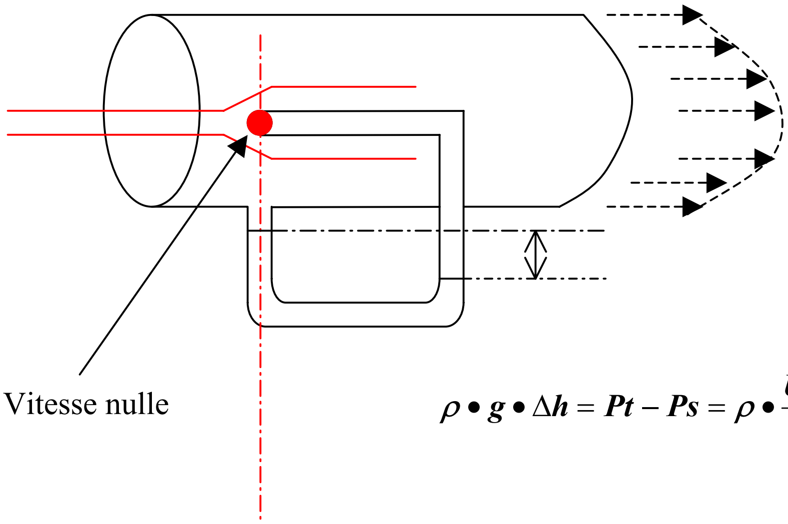
Application de Bernoulli

1) mesure de vitesse -> tube de Pitot
2) mesure de débit

Tube de Pitot

1) mesure de vitesse :

$$Qv = \iint_{(s)} U(s) \bullet ds$$



$$\rho \bullet g \bullet \Delta h = Pt - Ps = \rho \bullet \frac{U^2}{2}$$

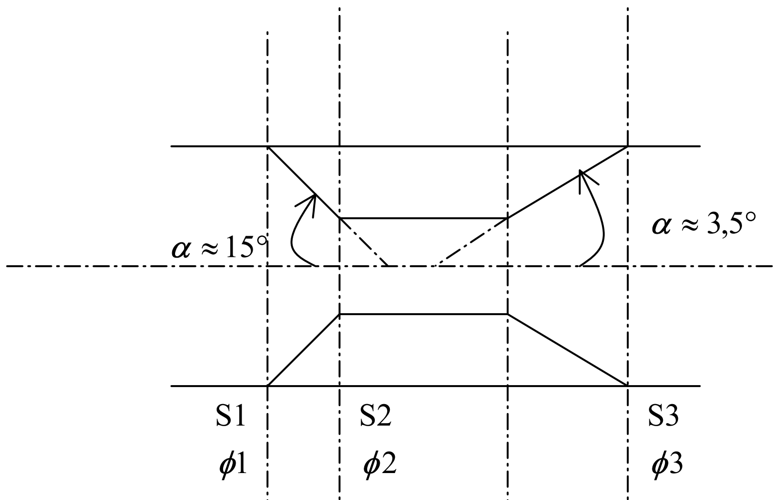
R.C.

$$\underbrace{Ps + \rho \bullet \frac{U^2}{2}}_{\text{Pt (pression totale)}} + \rho \bullet g \bullet z = \underbrace{P's + \rho \bullet \frac{U'^2}{2}}_{\text{statique}}$$

Pt (pression totale) statique

2) mesure de débit :

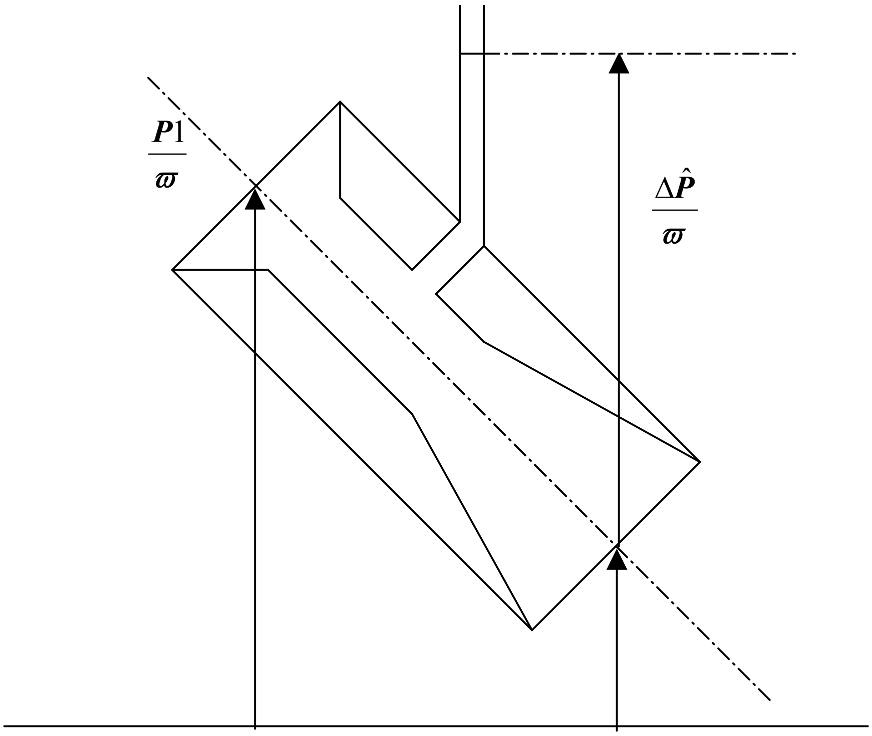
- venturi
- diaphragme
- tuyère



- Néglige la perte en charge
- Problème : chère, volumineux, étalonnage

$$P1 + \rho \bullet \frac{U_1^{\,2}}{2} + \varpi \bullet z_1 = P_2 s + \rho \bullet \frac{U_2^{\,2}}{2} + \varpi \bullet z_2$$

$\hat{P}1$ $\hat{P}2$



$$\rho \bullet \frac{U_1^{\ 2}}{2} - \rho \bullet \frac{U_2^{\ 2}}{2} + \varpi \bullet z_1 = \hat{P}_2 - \hat{P}_1$$

qv = Sn • Un = S2 • U2 d'où $U1 = U2 \bullet \frac{S2}{S1}$

$$\rho \bullet \frac{U_1^{\ 2}}{2} = \rho \bullet \frac{U_2^{\ 2}}{2} \bullet \left(\frac{S2}{S1}\right)^2$$

substituer U_1

$$\left[\left(\frac{S2}{S1}\right)^2 - 1\right] \bullet \rho \bullet \frac{U_2^{\ 2}}{2} = \Delta\hat{P}$$

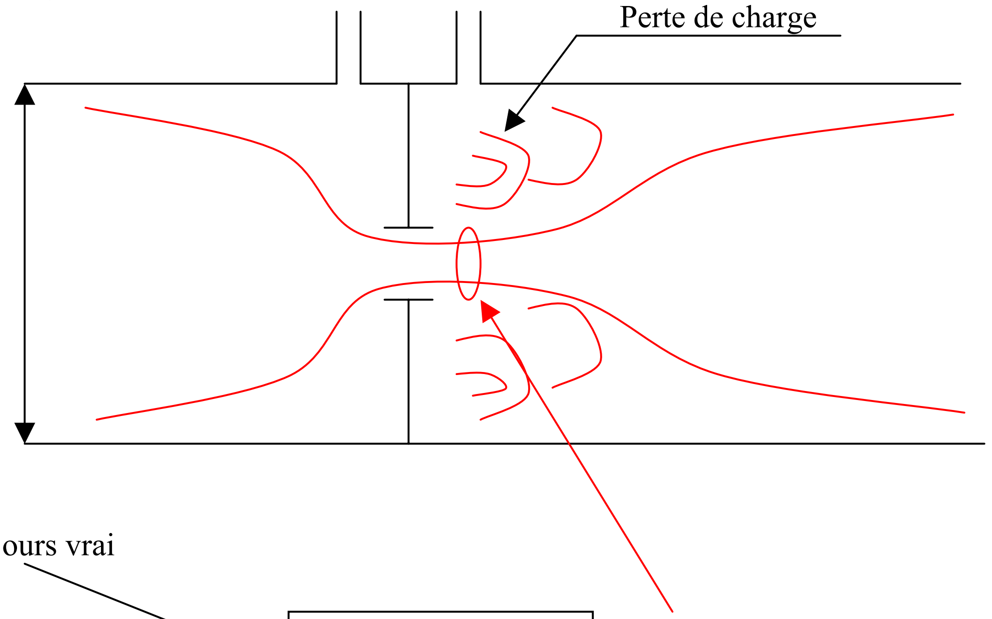
$$\Delta\hat{P} = P_1 - P_2$$

$$U_2 = \sqrt{\frac{2 \bullet \Delta\hat{P}}{1 - \left(\frac{S2}{S1}\right)^2}}$$

$$Qv = \frac{\pi \bullet \phi_2^2}{4} \bullet \sqrt{\frac{2 \bullet \Delta\hat{P}}{\rho \bullet \left(1 - \left(\frac{\phi_2}{\phi_1}\right)^4\right)}}$$

Coefficient de venturi

Le diaphragme:



Pas toujours vrai

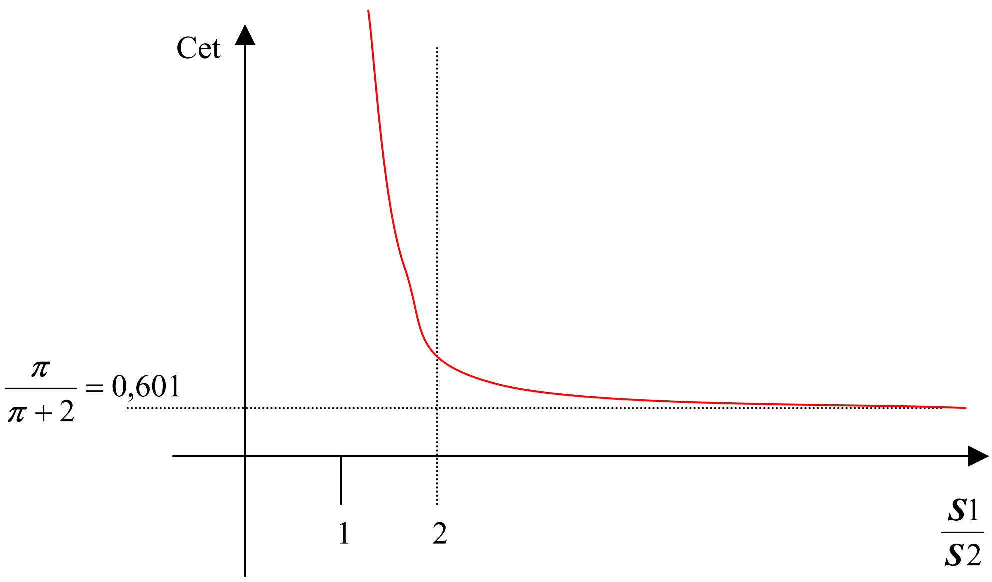
$$\frac{Sc}{S2} = \frac{\pi}{\pi + 2} = 0,601$$

Sc section contractée

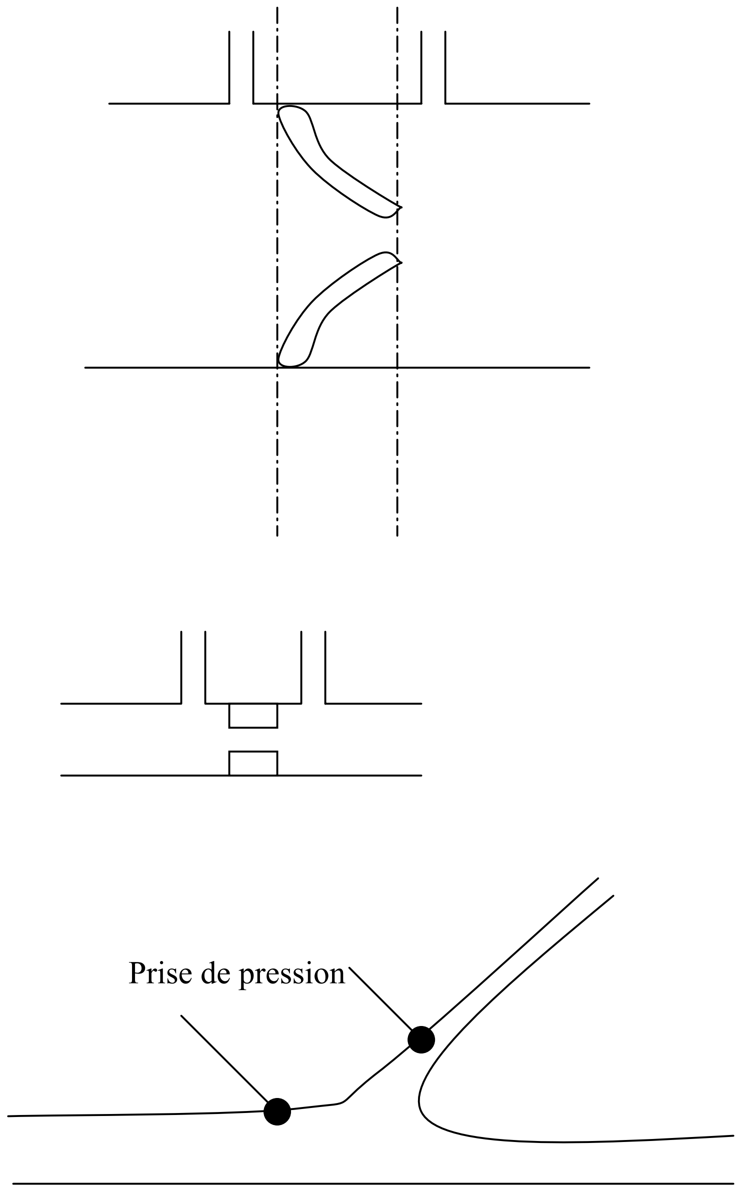
Si bernouilli :

$$Qv = \text{Cet} \bullet \frac{\pi \bullet \phi_2^2}{4} \bullet \sqrt{\frac{2 \bullet \Delta \hat{P}}{\rho \bullet \left(1 - \left(\frac{\phi_2}{\phi_1}\right)^4\right)}}$$

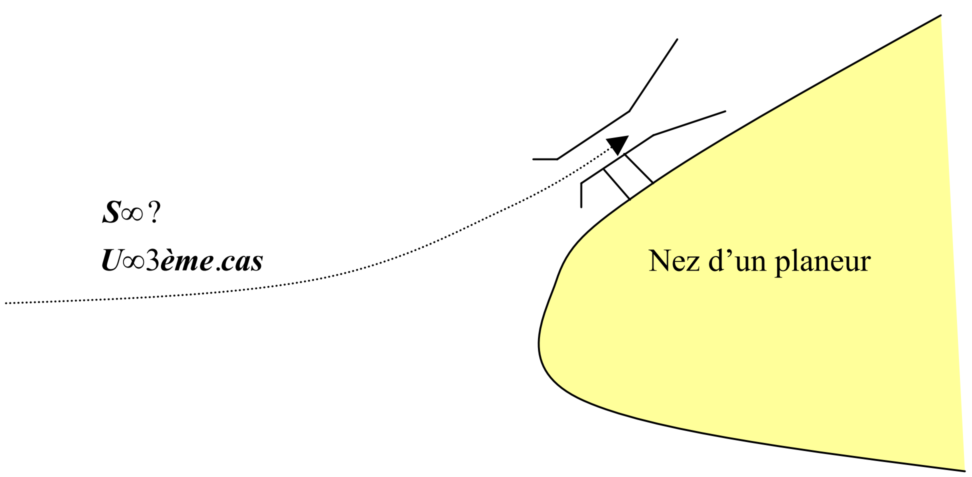
Cet -> Coeff d'étalonnage (Cet < 1)



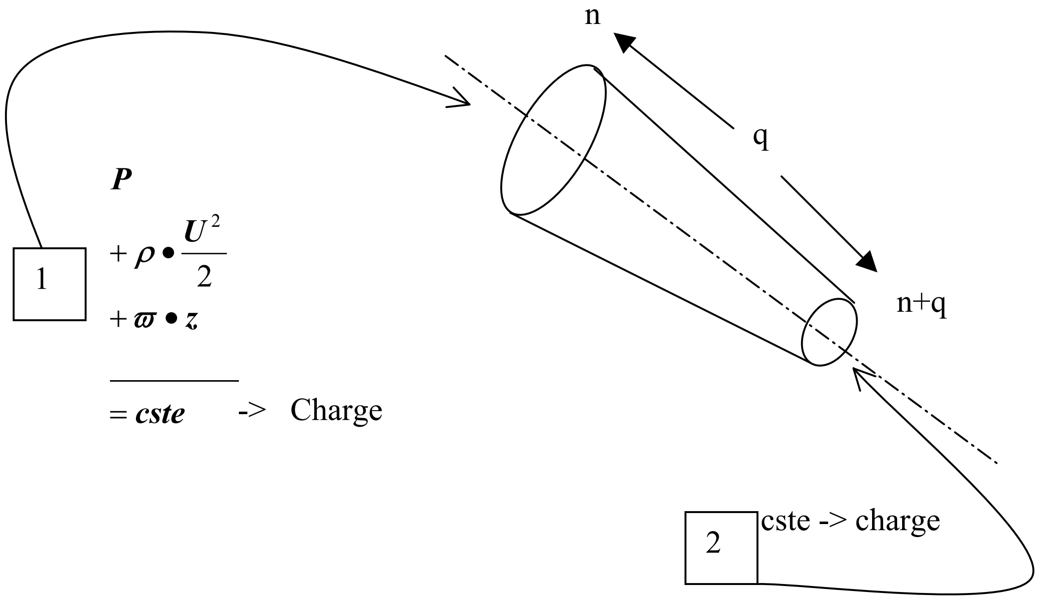
Exemple de tuyères :



3ème cas d'adaptation :



La perte de charge singulière :



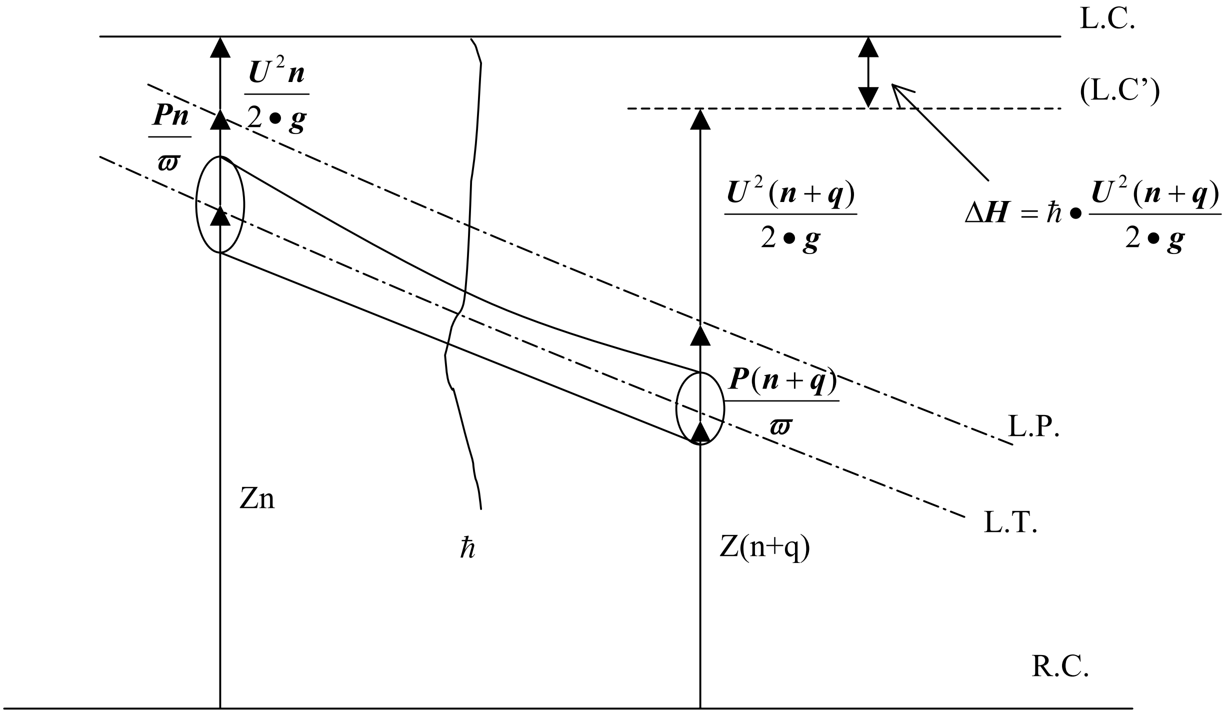
$$P_n + \rho \bullet \frac{U^2{}_n}{2} + \varpi \bullet z_n = P_{n+q} + \rho \bullet \frac{U^2{}_{n+q}}{2} + \varpi \bullet z_{n+q} + \Delta P_n^{\,n+q}$$

singularité : { Sn ; Sn+q} perte de charge sous forme de pression

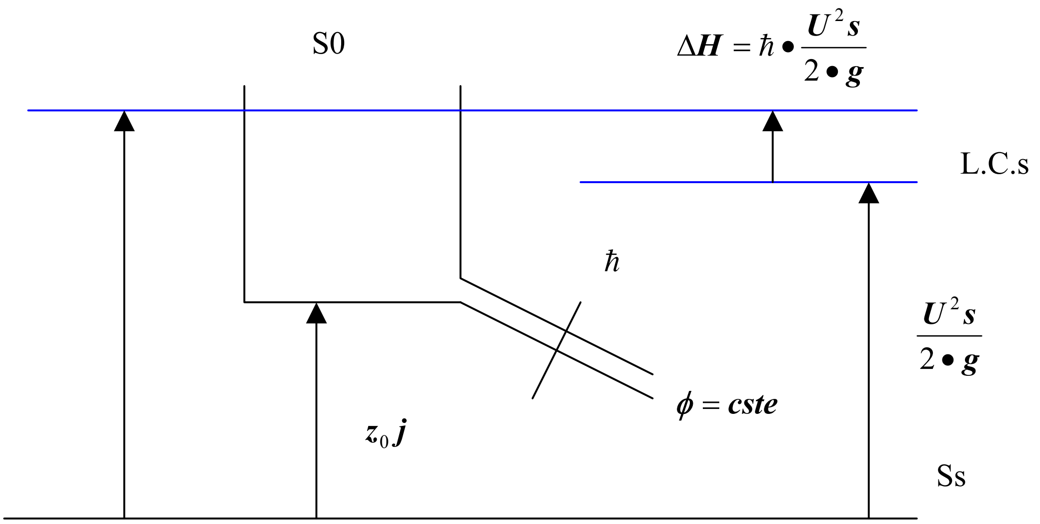
coefficient de perte de charge singulière

$\hbar \bullet \rho \bullet \frac{U^2 n}{2}$ hauteur connu avant la singularité

dzéta

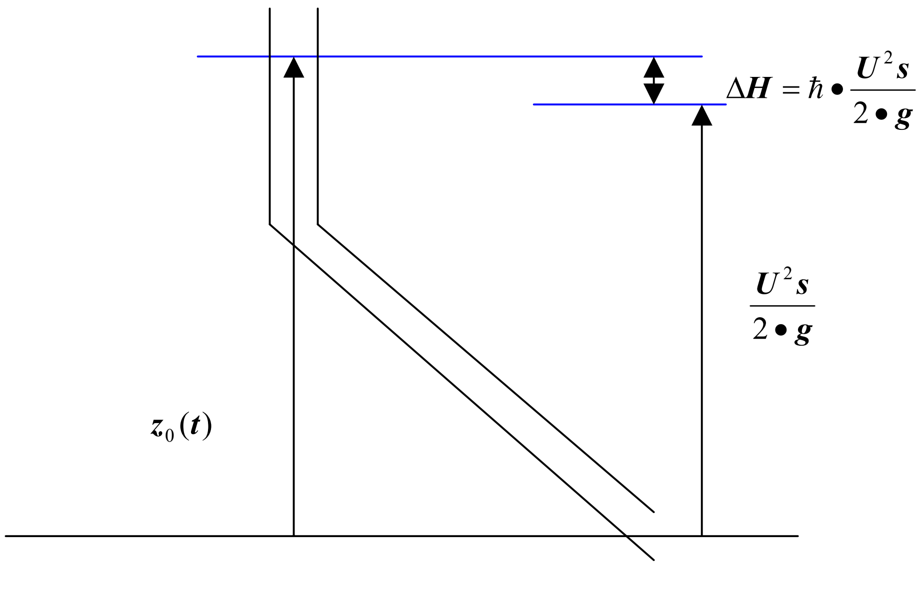


Rôle de la perte de charge sur une vidange :



Torichelli : $Us_{(t)} = \sqrt{2 \bullet g \bullet z_{(t)}}$

$$z_{(t)} = \left(\frac{U^2 s}{2 \bullet g}\right)_{(t)} + \hbar \bullet \left(\frac{U^2 s}{2 \bullet g}\right)_{(t)} \Rightarrow z_{(t)} = \left(\frac{U^2 s}{2 \bullet g}\right)_{(t)} \bullet (1 + \hbar)$$



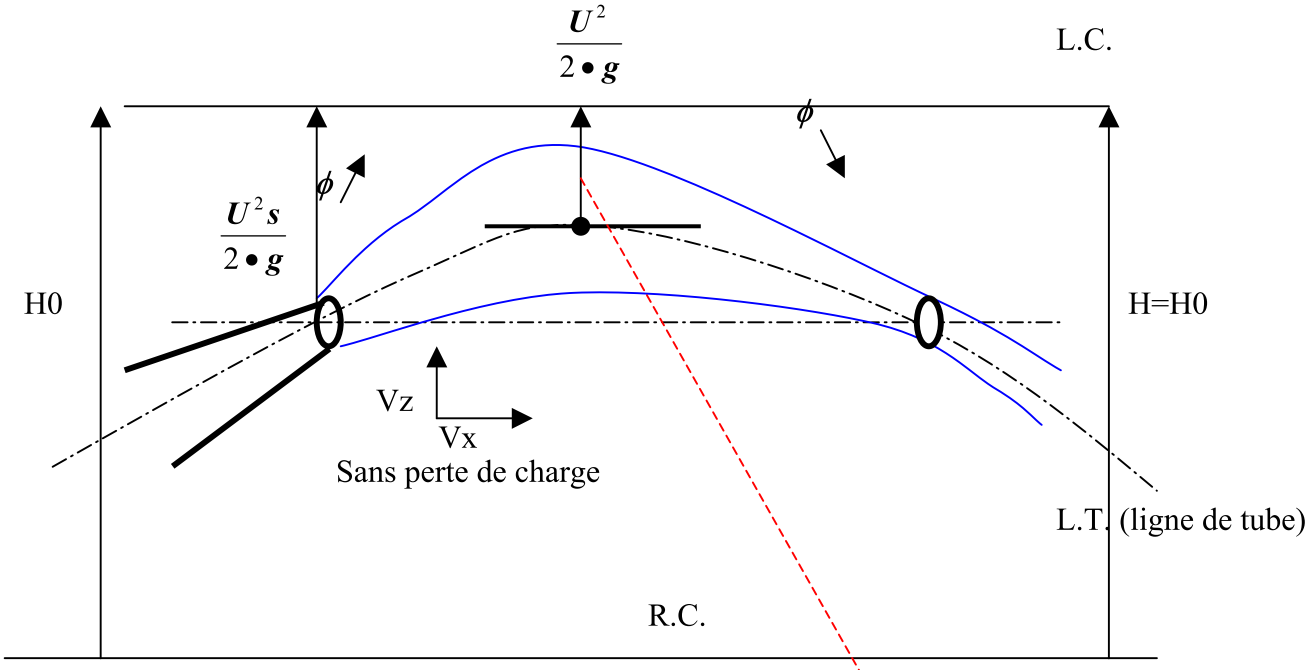
Us = ?

$$-\frac{dv}{dt} = Ss \bullet \sqrt{\frac{2 \bullet g \bullet z}{1 + \hbar}} \quad \text{avant : } -\frac{dv}{dt} = S1 \bullet \sqrt{2 \bullet g \bullet z}$$

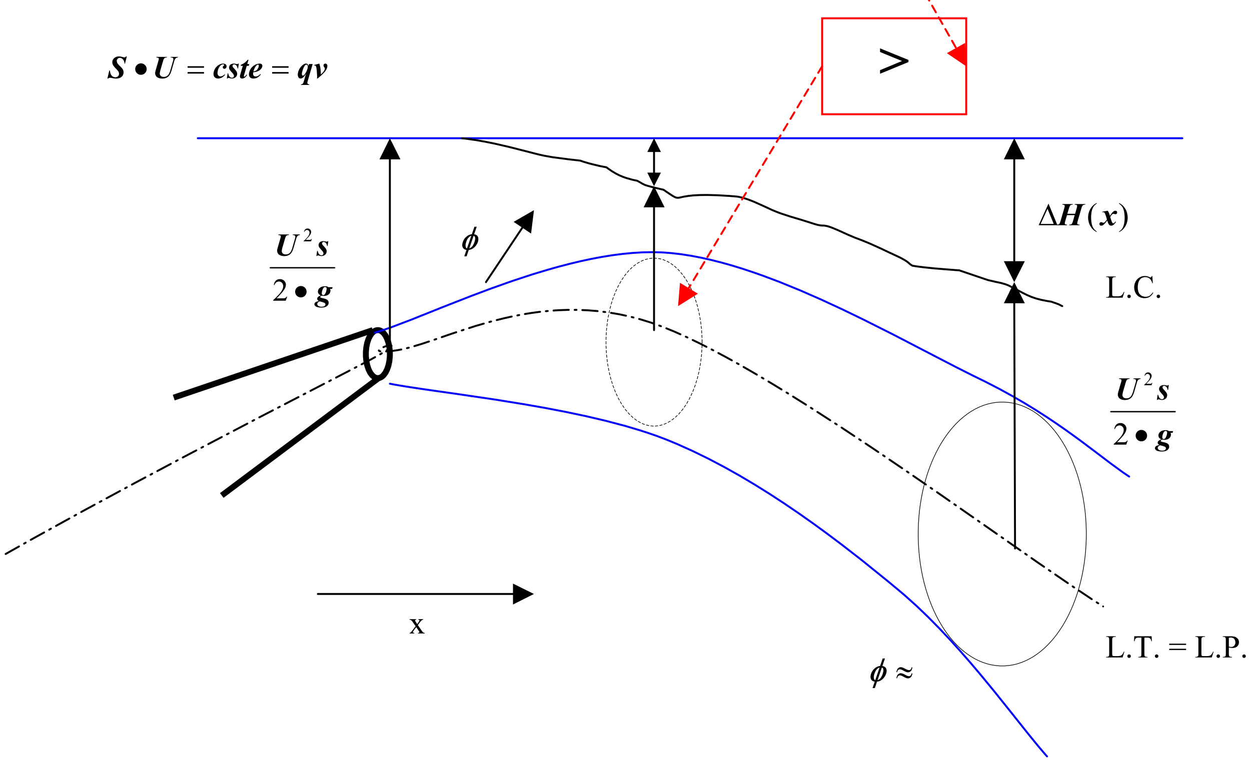
avec perte de charge : t avec perte de charge = t sans perte de charge $\bullet \sqrt{1+\hbar}$

3ème cas d'adaptation et dpc :

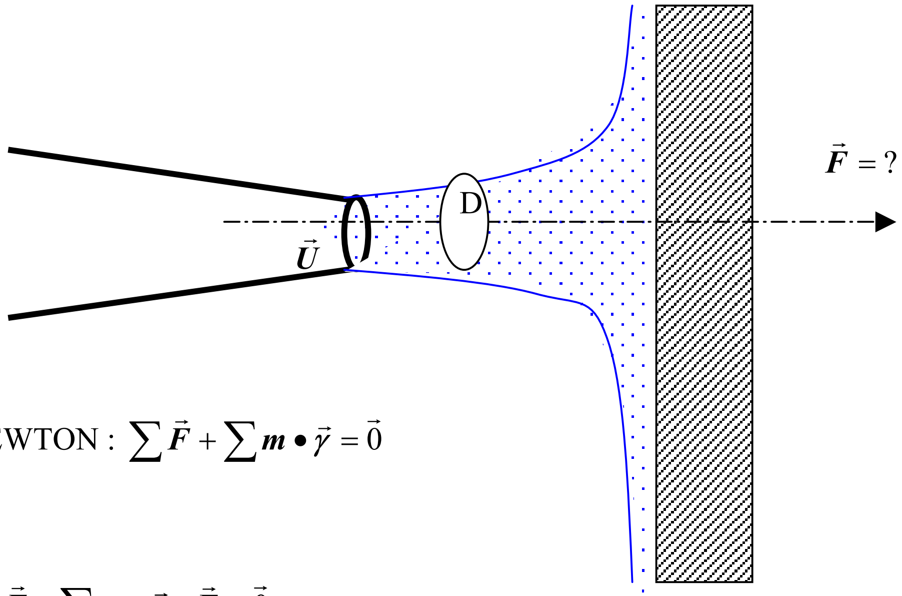
sans perte de charge :



$S \bullet U = cste = qv$



Théorème d'EULER (mécanique des fluides) sur les quantités de mvt :



NEWTON : $\sum \vec{F} + \sum m \bullet \vec{\gamma} = \vec{0}$

$\sum \vec{F} + \sum m \bullet \vec{\gamma} + \vec{\Gamma} = \vec{0}$

$\neq 0$

comment déterminer $\vec{\Gamma}$? : la méthode de RAYLEIGH

depend de $U; \rho; D$ $\quad m \bullet \gamma : M / s^{-2}$

$M^1 \bullet L^1 \bullet T^{-2} = \left[U^\alpha \bullet \rho^\beta \bullet D^\gamma\right]$

$$= \underbrace{\left[M^0 \bullet L^1 \bullet T^{-1}\right]^\alpha}_{\mathbf{U}} \bullet \underbrace{\left[M^1 \bullet L^{-3} \bullet T^0\right]^\beta}_{\rho} \bullet \underbrace{\left[M^0 \bullet L^1 \bullet T^0\right]^\gamma}_{\mathbf{D}}$$

$\beta = 1$

$L \rightarrow 1 = \alpha - 3\beta + \gamma$

$T \rightarrow -2 = -\alpha$

$U = \frac{dm}{dt} \bullet \vec{U}$

qm

$\rho \bullet U \bullet D^2 \bullet U$

$\rho \bullet U \bullet S$

$\rho \bullet qv$

qm

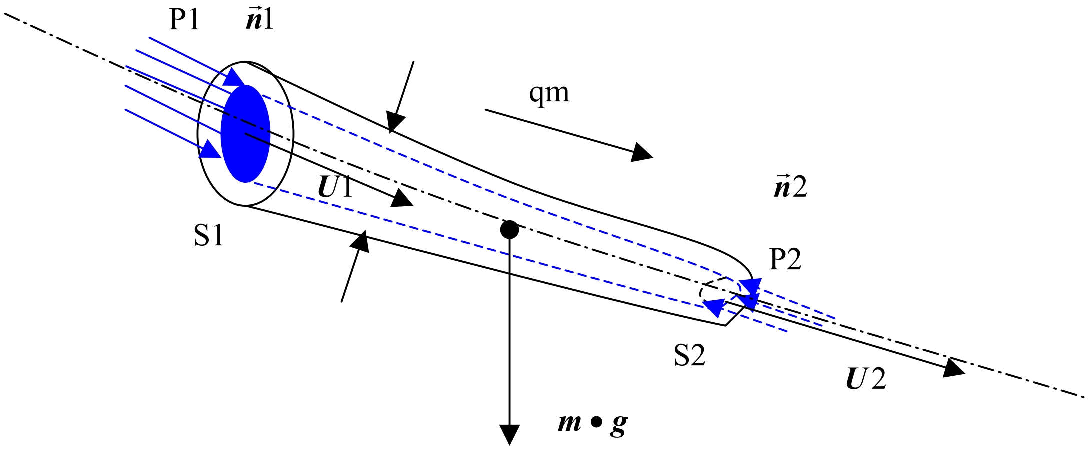
$\sum \vec{F} + \sum m \bullet \vec{\gamma} + \sum \frac{dm}{dt} \bullet \vec{U} = \vec{0}$

$m \bullet \frac{d\vec{u}}{dt} + \vec{U} \bullet \frac{dm}{dt} \rightarrow \frac{d}{dt} \bullet \left[m \bullet \vec{U}\right]$

$\mathbf{qm} \rightarrow \frac{dm}{dt}$

Application aux écoulements :

- **Masse volumique constante.**
- **Débit constant.**



$\sum \vec{F} \rightarrow \left[m \bullet \vec{g} + P1 \bullet S1 \bullet \vec{n}1 - P2 \bullet S2 \bullet \vec{n}2 + \sum i\vec{F}_{paroi} \right]$

.... +

$\sum m \bullet \vec{\gamma} \rightarrow [ecoulement....permanent \Rightarrow ..... = 0.........]$

.... +

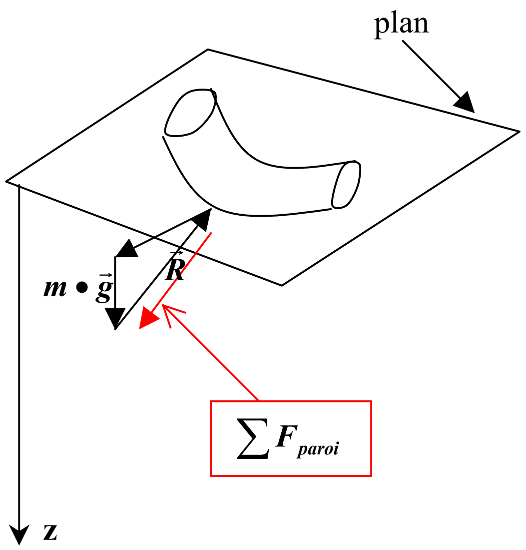
$\sum \frac{dm}{dt} \bullet \vec{U} \rightarrow \left[qm \bullet \vec{U}1 - qm \bullet \vec{U}2 \right]$

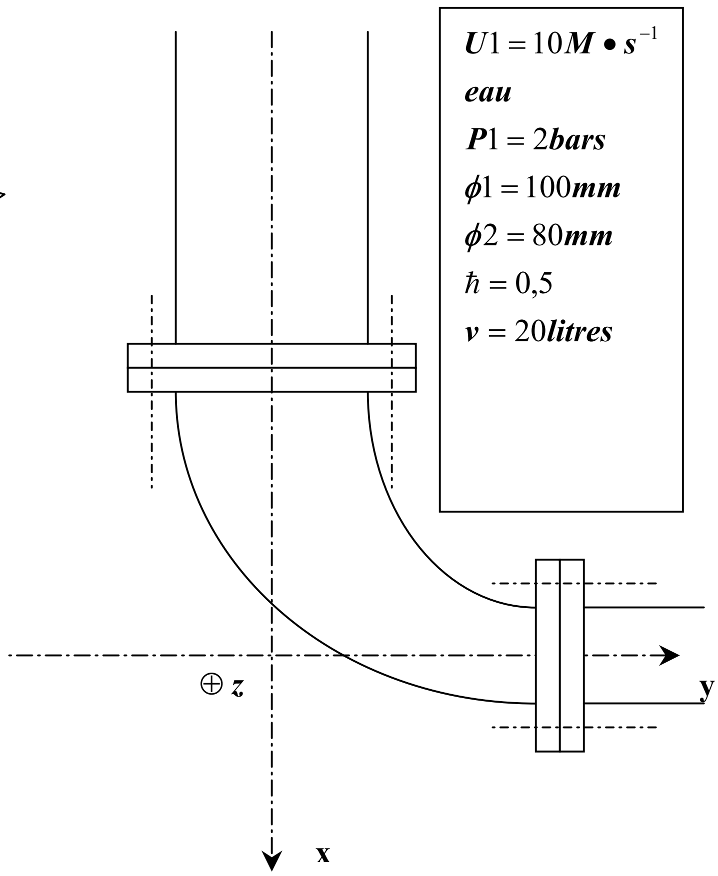
.... =

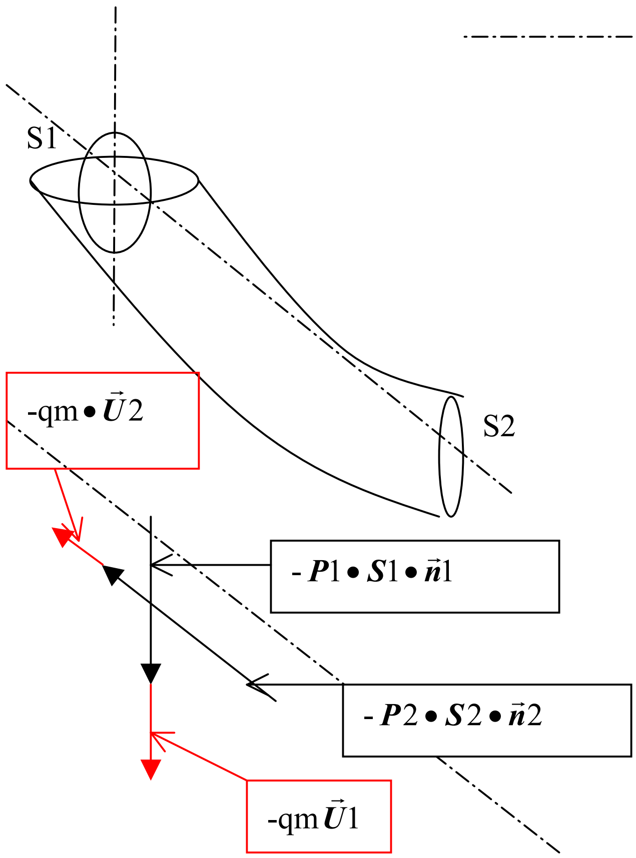
....$\vec{0}$

P1, P2 effectives.

Exercice :







$$S = \frac{\pi \bullet D^2}{4}$$

$$S1 \bullet U1 = S2 \bullet U2 \Rightarrow U2 = \frac{S1}{S2} \bullet U1 = 15{,}6 M \bullet s^{-2}$$

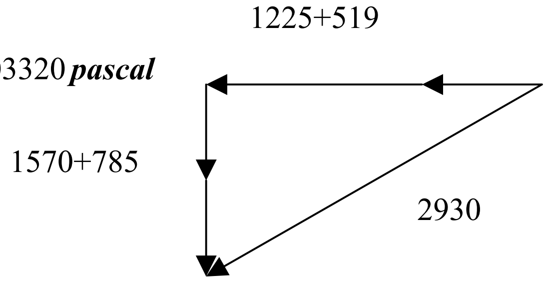
$$\frac{U_1^2}{2 \bullet g} + z_1 + \frac{P_1}{\rho \bullet g} = \frac{U_2^2}{2 \bullet g} + z_1 + \frac{P_2}{\rho \bullet g} + \hbar \bullet \frac{U^2{}_2}{2 \bullet g}$$

$$P_2 = P_1 - \Delta(\rho \bullet \frac{U^2}{2}) - \Delta P$$

$$\hbar \bullet \rho \bullet \frac{U_1^2}{2}$$

ρ = 1000 pour l'eau

$$P_2 = 210^5 + 50000 - 25000 - 500 \bullet 15{,}6^2 = 103320\, \textit{pascal}$$



$P_1 \bullet S_1 = 1570N$

$P_2 \bullet S_2 = 519N$

$qm \bullet U_1 = 785N$

$qm \bullet U_2 = 1224N$

$m \bullet g = 200N$