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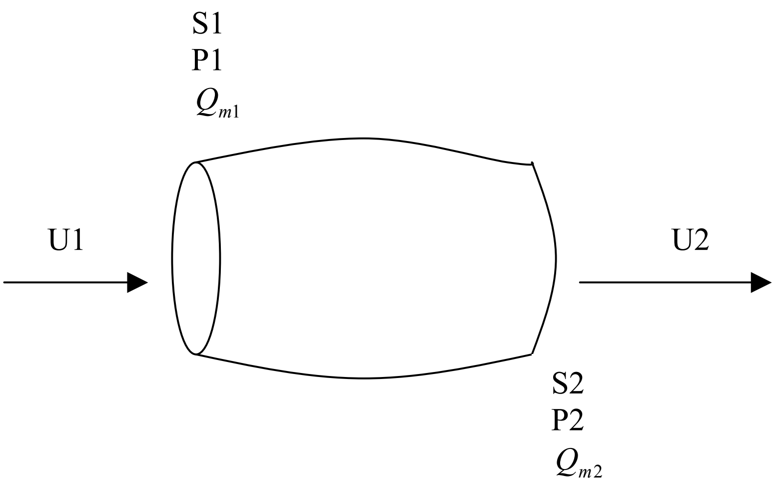
Un réacteur :



Pression effective

$$\boxed{Q_{m1} \bullet \vec{u}_1 + P_1 \bullet S_1 \bullet \vec{n}_1 - Q_{m2} \bullet \vec{u}_2 + P_2 \bullet S_2 \bullet \vec{n}_2 + \underbrace{\Sigma_1 \overrightarrow{F_P}}_{\text{poussée}} = \vec{0}}$$

1% (effet de fusée) débit de carburant

$$Q_{m2} = Q_{m1} + \cancel{Q_{mc}}$$

$$Q_m = \rho_1 \bullet S_1 \bullet U_1 = \rho_2 \bullet S_2 \bullet U_2$$

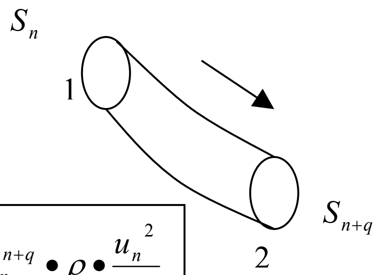
$$\boxed{Q_m \bullet (\vec{u}_1 - \vec{u}_2) = -\Sigma_1 \overrightarrow{F_P}}$$

$$\frac{P_1}{r \bullet T_1} \qquad \frac{P_2}{r \bullet T_2}$$

C'est la dilatation des gaz qui produit la poussée

Les pertes de charges

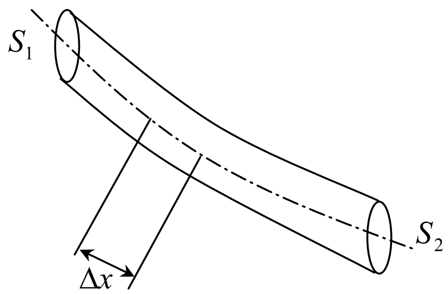
Coefficient de perte de charges :
- singulière : ζ
- linéique : λ



Hypothèse :

- φ constante u $\neq$ 0
- écoulement permanent
- solide

$$\boxed{\Delta P_n^{n+q} = \zeta_n^{n+q} \bullet \rho \bullet \frac{u_n^{\ 2}}{2}}$$



$$\rho \bullet \frac{U_1^{\ 2}}{2} + \rho \bullet g \bullet z_1 + P_1 = \rho \bullet \frac{U_2^{\ 2}}{2} + \rho \bullet g \bullet z_2 + P_2$$

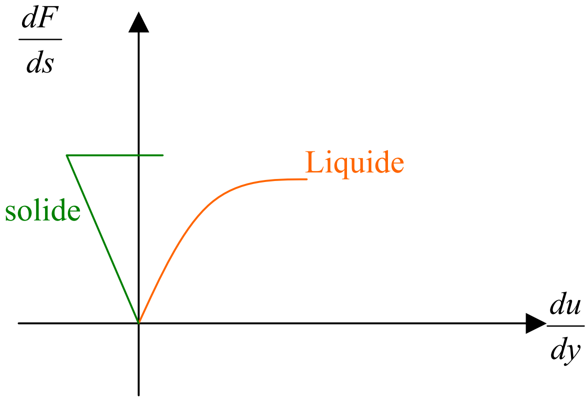
$$P_1 - P_2 = \Delta P_1^{\ 2} = \zeta_1^{\ 2} \bullet \rho \bullet \frac{U_1^{\ 2}}{2}$$

$$P_1 - P_2 = \Delta P_1^{\ 2} = \lambda \bullet \rho \bullet \frac{U_1^{\ 2}}{2} \bullet \frac{\Delta x}{D}$$

Perte de charge linéique

1) Les écoulements laminaires :

- les lois de Newton
- Les lois de Poiseuilles (dans un tuyau laminaire)
- Les lois de Reynolds : $\mathrm{Re} = \rho \bullet \frac{u \bullet D}{\mu}$



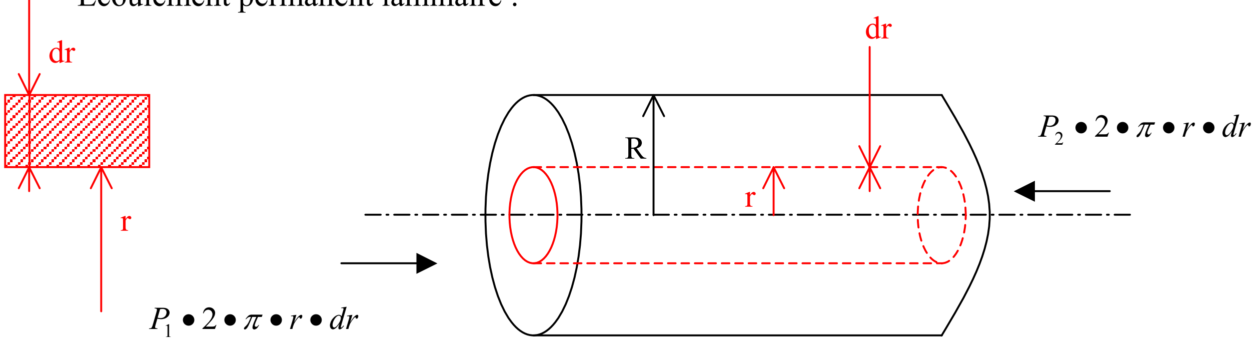
Newton

$$\tau = \frac{\partial F}{\partial s} = \mu \bullet \frac{\partial u}{\partial y}$$

$\mathrm{Re} = \frac{\rho \bullet u \bullet D}{\mu}$ Re < 2000 => écoulement laminaire

Re > 2000 => écoulement turbulent.

Ecoulement permanent laminaire :



- Fp pression
- Fv viscosité

$\mathrm{Fp} = \Delta p \bullet 2 \bullet \pi \bullet r \bullet dr$ $\quad \mathrm{Fvi} = \tau \overbrace{2 \bullet \pi \bullet r \bullet \Delta x}^{\text{surface}} = \mu \bullet \frac{du}{dr} \bullet 2 \bullet \pi \bullet r \bullet \Delta x$

Fv ext = Fvi + dFvi

$$\frac{dFvi}{dr} = 2 \bullet \pi \bullet \mu \bullet \Delta x \bullet \frac{d}{dr} \bullet (r \bullet \frac{du}{dr}) \bullet dr$$

$$\Delta P \bullet 2 \bullet \pi \bullet r \bullet \cancel{dr} = 2 \bullet \pi \bullet \mu \bullet \Delta x \bullet \frac{d}{dr} \bullet (r \bullet \frac{du}{dr}) \bullet \cancel{dr}$$

$$\frac{d}{dr} \bullet (r \bullet \frac{du}{dr}) = \frac{1}{\mu} \bullet \frac{\Delta P}{\Delta x} \bullet r$$

1ère intégration

$$\cancel{r} \bullet \frac{du}{\cancel{dr}} = (\frac{1}{2 \bullet \mu} \bullet \frac{\Delta P}{\Delta x} \bullet r^2 + \frac{C1}{r}) \bullet dr$$

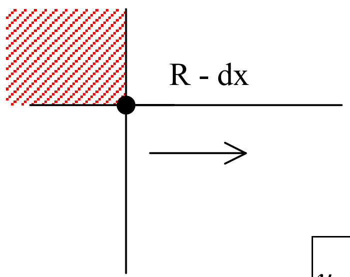
2ème intégration

$$u_{(r)} = \frac{1}{4 \bullet \mu} \bullet \frac{\Delta P}{\Delta x} \bullet r^2 + C1 \bullet \ln r + C2$$

si r = 0 => C1 =0 vitesse $\longrightarrow$ $+\infty$ pas possible !

condition d'adhérence :

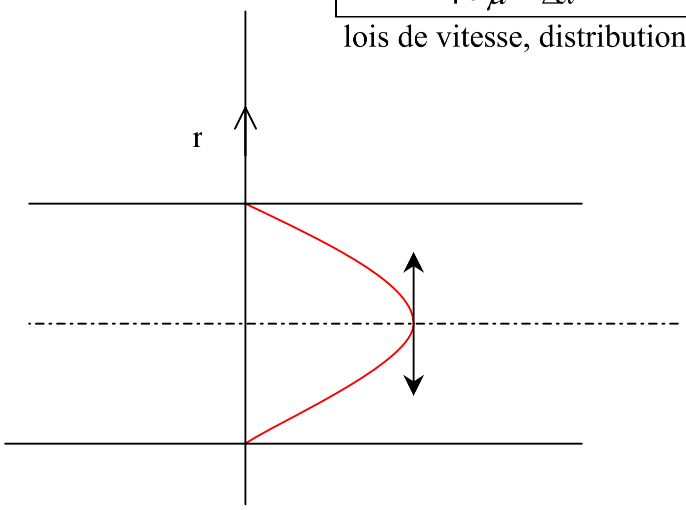
u = 0 si r = R



1ère lois de poiseuille :

$$\boxed{u_{(r)} = -\frac{1}{4 \bullet \mu} \bullet \frac{\Delta P}{\Delta x} \bullet (R^2 - r^2)}$$

lois de vitesse, distribution de vitesse.



2ème lois de poiseuille :

$$dq_r = u_{(r)} \bullet 2 \bullet \pi \bullet r \bullet dr = -\frac{1}{4 \bullet \mu} \bullet \frac{\Delta P}{\Delta x} \bullet (R^2 - r^2) \bullet 2 \bullet \pi \bullet r \bullet dr$$

$qv = \int_0^r dqv$ d'où $\boxed{q_r = u_{(r)} \bullet 2 \bullet \pi \bullet r \bullet dr = -\frac{1}{8 \bullet \mu} \bullet \frac{\Delta P}{\Delta x} \bullet R^4}$ débit

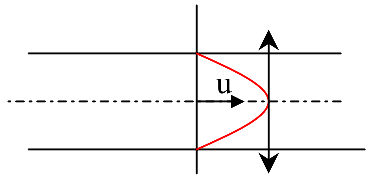
3ème lois de poiseuille :
relation entre poiseuille et l'équation général de perte de charge :

$\Delta P = \lambda \bullet \rho \bullet \frac{u^2}{2} \bullet \frac{\Delta x}{D}$ $U = \frac{qv}{\rho \bullet s}$ vitesse débitante :

$\Delta P = -\frac{qv \bullet 8 \bullet \mu \bullet \Delta x}{\pi \bullet R^4}$ pour r = 0

convention de poiseuille

$U = -\frac{1}{8 \bullet \mu} \bullet \frac{\Delta P}{\Delta x} \bullet R^2$ $U = \frac{1}{2} \bullet u$ quand r = 0



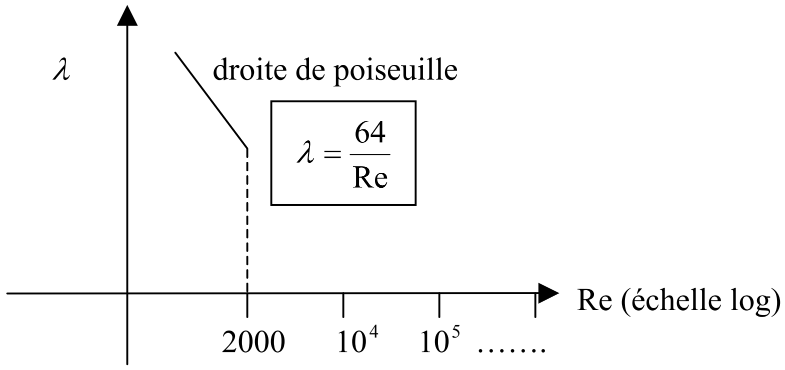
$\lambda \bullet \rho \bullet \frac{u^{\cancel{2}}}{2} \bullet \frac{\cancel{\Delta x}}{\cancel{D}} = \frac{\cancel{u} \bullet 8 \bullet \mu \bullet \cancel{\Delta x}}{R^{\cancel{2}}}$ $\lambda = \frac{\mu \bullet 8 \bullet 2 \bullet 2 \bullet 2}{\rho \bullet u \bullet D}$ d'où $\boxed{\lambda = 64 \bullet \mathrm{Re}^{-1}}$

$\boxed{2 \bullet \mathrm{R}}$

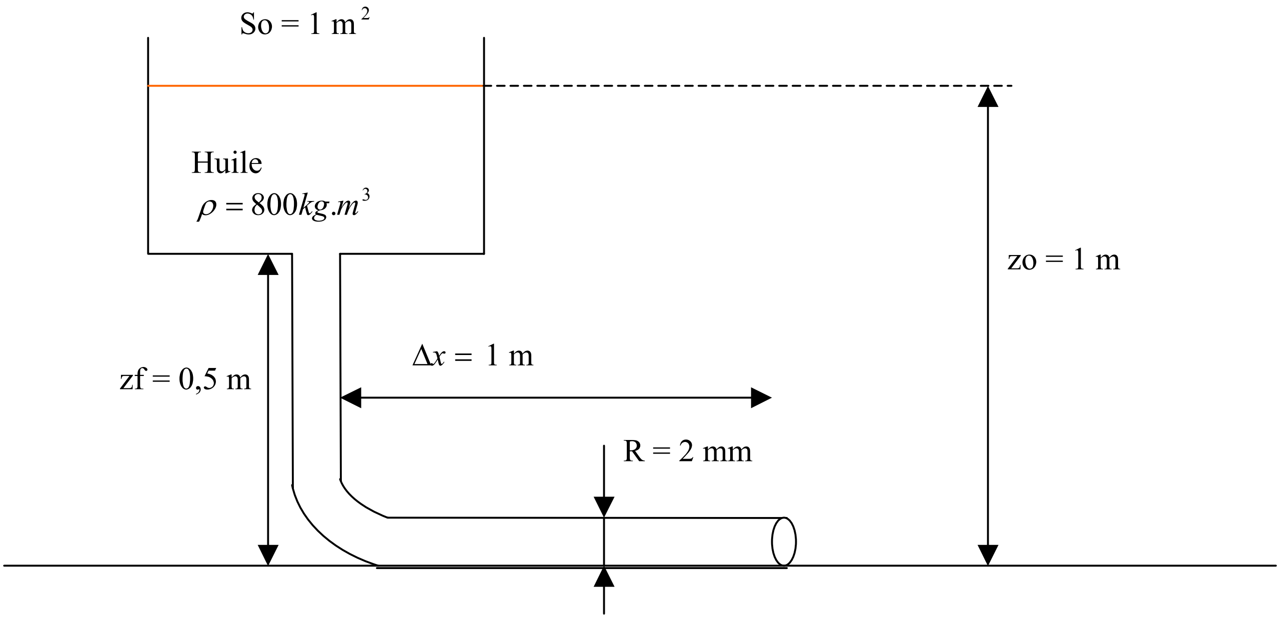
3ème lois de poiseuille

U

$\Delta P = -\frac{\cancel{qv} \bullet 8 \bullet \mu \bullet \Delta x}{\pi \bullet R^{\cancel{4}\,2}}$



Méthode de résolution de problèmes laminaires



μ = 10 m pp (milli poiseuille (système S.I.) ou des $\dfrac{N \bullet m^{-2}}{s^{-1}}$)

qv = ?

$$\cancel{Po} + \rho \bullet \frac{U_o^{\ 2}}{2} + \varpi \bullet z_o = \cancel{Ps} + \rho \bullet \frac{U_s^{\ 2}}{2} + \varpi \bullet \cancel{z_s} + \lambda \bullet \rho \bullet \frac{u^2}{2} \bullet \frac{\Delta x}{D}$$

on néglige la variation de pression atmosphérique $z_s = 0$

suposons Re < 2000 => $\lambda = \dfrac{64}{\mathrm{Re}}$

On néglige So > Ss

$$\varpi \bullet z_o = \rho \bullet \frac{U_s^{\ 2}}{2} + \lambda \bullet \rho \bullet \frac{u^2}{2} \bullet \frac{\Delta x}{D}$$

sans perte de charge

$Us\max \rightarrow u_s = \sqrt{2 \bullet g \bullet z}$ $\quad \mathrm{Re} = \dfrac{800 \bullet 4 \bullet 2 \bullet 10^{-3}}{10^{-2}} = 640$ on est certain d'être laminaire

$$\varpi \bullet z_o = \rho \bullet \frac{u_s^{\,2}}{2} + \frac{u_s \bullet 8 \bullet \mu \bullet \Delta x}{R^2}$$

$$\frac{\varpi}{400} = 800 \text{ x } 9{,}81 = 19{,}62$$

$$\frac{\rho}{2} = \frac{400}{400} = 1$$

$$\frac{8 \bullet \mu \bullet \Delta x}{R^2} = \frac{8 \times 10^{-2} \times 2}{(2 \times 10^{-3})^2} \times \frac{1}{400} = 50$$

x

$$\sqrt{b^2 - 4 \bullet a \bullet c} = 50{,}77$$

$\Delta = b^2 - 4 \bullet a \bullet c$

$\Delta > 0$ d'où 2 racines réelles

$$\text{x1} = \frac{-b + \sqrt{\Delta}}{2 \bullet a}$$

d'où $\mathrm{u}_s = 0{,}39 \text{ m.s}^{-1}$

$$\text{x1} = \frac{-b - \sqrt{\Delta}}{2 \bullet a}$$

réinjectons u_s dans l'équation :

$\varpi \bullet z_o = \rho \bullet \dfrac{U_s^{\,2}}{2} + \dfrac{u_s \bullet 8 \bullet \mu \bullet \Delta x}{R^2} = 400 \text{ x } 0{,}39^2 + 0{,}39 \text{ X } 500$ d'où $\rho \bullet \dfrac{U_s^{\,2}}{2}$ est négligeable devant $\dfrac{u_s \bullet 8 \bullet \mu \bullet \Delta x}{R^2}$

avec $0{,}4 \text{ m.s}^{-1}$ $\quad \mathrm{Re} = \dfrac{800 \times 0{,}4 \times 4 \times 10^3}{10^{-2}}$

Calcul du temps de vidange :

$$\varpi \bullet z_o = \frac{u_s \bullet 8 \bullet \mu \bullet \Delta x}{R^2}$$

$$-\frac{dv}{dt} = S_s \bullet u_s \qquad \mathrm{dv} = \mathrm{S}_o\,(\mathrm{z}) \bullet \mathrm{dz}$$

$1\ \mathrm{m}^2$

$$\mathrm{u}_s = \frac{\varpi \bullet z_o \bullet R^2}{8 \bullet \mu \bullet \Delta x}$$

$\mathrm{z}_o\,(\mathrm{t})$

$\mathrm{dz} = \mathrm{dz}_o\,(\mathrm{t})$

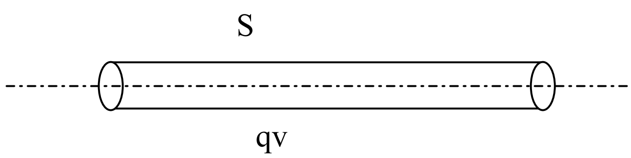
$$\int dt = \int_{z_o}^{z_f} -\frac{8 \bullet \mu \bullet \Delta x}{R^2} \bullet \frac{1}{z(dt)} \bullet dv \qquad \mathrm{t} = \mathrm{K} \bullet \ln\left(\frac{z_o}{z_f}\right)$$

$$-\mathrm{S}_o\,(\mathrm{z}) \bullet \frac{dz}{dt} = \frac{\varpi \bullet z_o \bullet R^2 \bullet S_s}{8 \bullet \mu \bullet \Delta x}$$

$$\mathrm{dt} = -\frac{S_o}{S_s}(z) \bullet \frac{8 \bullet \mu \bullet \Delta x}{\varpi \bullet R^2} \bullet z^{-1} \bullet dz \qquad \mathrm{t} = +\mathrm{K} \bullet \ln\left(\frac{z_o}{z_f}\right)$$

Méthode de résolution de problème turbulent

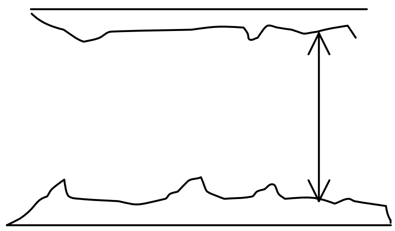
1) problème direct :



$$\Delta P = \lambda \bullet \rho \bullet \frac{u^2}{2} \bullet \frac{\Delta x}{D}$$

V = $\dfrac{qv}{S}$ $\rho = 1000 kg \cdot m^{-3}$ V =10 m.s^{-1} D = 50 mm $\Delta x = 10m$ $\mu = 1mPl$

Indice de rugosité selon Riman :



k$_s$ ou $\bar{\varepsilon} = \int_0^L \varepsilon(x) \bullet dx$

$\bar{\varepsilon} = 0{,}5$ mm => $\dfrac{k_s}{D} = \dfrac{0{,}5}{50} = 0{,}01$

Re = $\dfrac{1000 \times 10 \times 50 \times 10^{-3}}{10^{-3}} = 500000$

Lecture de λ sur la courbe liant λ avec Re

D'où $\Delta P = 0{,}038 \times \dfrac{1000}{2} \times 10^2 \times \dfrac{10}{50 \times 10^{-3}}$ (pascal) = 3,8 bar

J = $\dfrac{\Delta P}{\Delta x}$ perte de charge linéique = 0, 38 bar

2) problème indirect, ΔP débit ?