MATRICE

E de dim n $\mathscr{B} = (\vec{e}_1,.....\vec{e}_n)$ choisie

$\vec{v}_1 = a_{11} \bullet \vec{e}_1 + a_{21} \bullet \vec{e}_2 + ...a_{n1} \bullet \vec{e}_n$ composante d'un vecteur sur une base

$\vec{v}_2 = a_{12} \bullet \vec{e}_1 + a_{22} \bullet \vec{e}_2 + ...a_{n2} \bullet \vec{e}_n$

$\vdots$

$\vec{v}_p = a_{1p} \bullet \vec{e}_1 + a_{2p} \bullet \vec{e}_2 + ...a_{np} \bullet \vec{e}_n$

$$\vec{v}_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ : \\ a_{n1} \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ : \\ a_{n2} \end{pmatrix} \quad \vec{v}_p = \begin{pmatrix} a_{1p} \\ a_{2p} \\ : \\ a_{np} \end{pmatrix} \quad \vec{V} = \begin{pmatrix} a_{11} & a_{12} & .... & a_{1p} \\ a_{21} & a_{22} & .... & a_{2p} \\ : & : & a_{ij} & : \\ a_{n1} & a_{n2} & ... & a_{np} \end{pmatrix} \begin{matrix} \leftarrow \vec{e}_1 \\ \leftarrow \vec{e}_2 \\ \leftarrow \vec{e}_i \\ \leftarrow \vec{e}_n \end{matrix}$$

$$\begin{matrix} \uparrow & \uparrow & .... & \uparrow \\ \vec{v}_1 & \vec{v}_2 & .\vec{v}_j & \vec{v}_p \end{matrix}$$

E de dim n $\mathscr{B} = (\vec{e}_1,.....\vec{e}_p)$ choisie

E de dim n $\mathscr{B}' = (\vec{\varepsilon}_1,.....\vec{\varepsilon}_n)$ choisie

$E \xrightarrow{f} F$ f linéaire

Matrice de f :

$$\begin{pmatrix} a_{11} & ..... & a_{1p} \\ : & a_{ij} & : \\ a_{n1} & ... & a_{np} \end{pmatrix} \begin{matrix} \leftarrow \vec{\varepsilon}_1 \\ \leftarrow \vec{\varepsilon}_i \\ \leftarrow \vec{\varepsilon}_n \end{matrix}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ f(\vec{e}_1) & f(\vec{e}_j) & f(\vec{e}_p) \end{matrix}$$

Ex :

$f \quad \mathbb{R}^4 \longrightarrow \mathbb{R}^4$

$(x, y, z, t) \mapsto (x + y + z ; x + y + t ; t - z)$

$\mathscr{B} = [(1 , 0 , 0 , 0) ; (0 , 1 , 0 , 0) ; (0 , 0 , 1 , 0) ; (0 , 0 , 0 , 1)]$

$\mathscr{B}' = [(1 , 0 , 0) ; (0 , 1 , 0) ; (0 , 0 , 1)]$

$f[(1 , 0 , 0 , 0)] = (1 , 1, 0)$

$f[(0 , 1 , 0 , 0)] = (1 , 1, 0)$

$f[(0 , 0 , 1 , 0)] = (1 , 0, -1)$

$f[(0 , 0 , 0 , 1)] = (0 , 1, 1)$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

ex : $f\ \mathcal{P}_3 \rightarrow \mathcal{P}_3$

$\mathcal{P} \rightarrow \mathcal{P}'$ $\mathcal{B}$ = (1 , x , x^2, x^3) base canonique de $\mathcal{P}_3$ $\mathcal{B}' = ?$

$f(1) = 0$
$f(x) = 1$
$f(x^2) = 2x$
$f(x^3 1) = 3x^2$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \leftarrow 1 \\ \leftarrow x \\ \leftarrow x^2 \\ \leftarrow x^3 \end{matrix}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ f(1) & f(x) & f(x^2) & f(x^3) \end{matrix}$$

$\forall \vec{v} \in E \quad \vec{v} = x_1 \bullet \vec{e}_1 + ...x_p \bullet \vec{e}_p$

$f(\vec{v}) = f(x_1 \bullet \vec{e}_1 + ...x_p \bullet \vec{e}_p)$ f linéaire $\Rightarrow$ $f(\vec{v}) = x_1 \bullet f(\vec{e}_1) + ...x_p \bullet f(\vec{e}_p)$

$$= \begin{matrix} x_1[a_{11}\vec{\varepsilon}_1 + a_{21}\vec{\varepsilon}_2 + ... + a_{n1}\vec{\varepsilon}_n] \\ + \\ \vdots \\ x_p[a_{1p}\vec{\varepsilon}_1 + a_{2p}\vec{\varepsilon}_2 + ... + a_{np}\vec{\varepsilon}_n] \end{matrix} = \begin{matrix} (a_{11} \bullet x1 + a_{12} \bullet x_2 .... + a_{1p} \bullet x_p) \bullet \vec{\varepsilon}_1 \\ + \\ \vdots \\ (a_{n1} \bullet x1 + a_{n2} \bullet x_2 .... + a_{np} \bullet x_p) \bullet \vec{\varepsilon}_n \end{matrix}$$

$$\begin{matrix} y_1 = a_{11} \bullet x_1 + ...a_{1p} \bullet x_p \\ \vdots \\ y_n = a_{n1} \bullet x_1 + ...a_{np} \bullet x_p \end{matrix}$$

Y = A X

$$\begin{matrix} x_1 \\ : \\ x_p \end{matrix}$$

$$\begin{matrix} a_{11} & ...... & a_{1p} \\ : & ..:......... & : \\ a_{n1} & ...... & a_{np} \end{matrix} \qquad \begin{matrix} y_1 = a_{11} \bullet x_1 + ...a_{1p} \bullet x_p \\ \vdots \\ y_n = a_{n1} \bullet x_1 + ...a_{np} \bullet x_p \end{matrix}$$

ex : $f\ \mathcal{P}_3 \rightarrow \mathcal{P}_3$

$\mathcal{P} \rightarrow \mathcal{P}' = ?$ $\mathcal{B}$ = (1 , x , x^2, x^3) base canonique de $\mathcal{P}_3$

P(x) = 13 – 7x + 3 x^2 + 5 x^3 P'(x) = ?

$$P(x) = \begin{pmatrix} 13 \\ -7 \\ 3 \\ 5 \end{pmatrix}$$

$$\begin{matrix} 0 & 1 & 0 & 0 & \leftarrow 1 \\ 0 & 0 & 2 & 0 & \leftarrow x \\ 0 & 0 & 0 & 3 & \leftarrow x^2 \\ 0 & 0 & 0 & 0 & \leftarrow x^3 \\ \uparrow & \uparrow & \uparrow & \uparrow & \\ f(1) & f(x) & f(x^2) & f(x^3) & \end{matrix}$$

P'(x) = 0 + (-7) (1) + (2 . 3) x + (3 . 5) x^2 = -7 + 6x + 15 x^2

f E -> F A g E -> F B f + g E -> F A + B λ f E -> F λ A

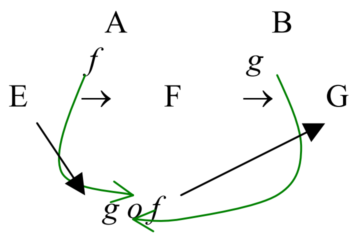
$f \quad E \xrightarrow{A} F$

$g \quad E \xrightarrow{B} F$

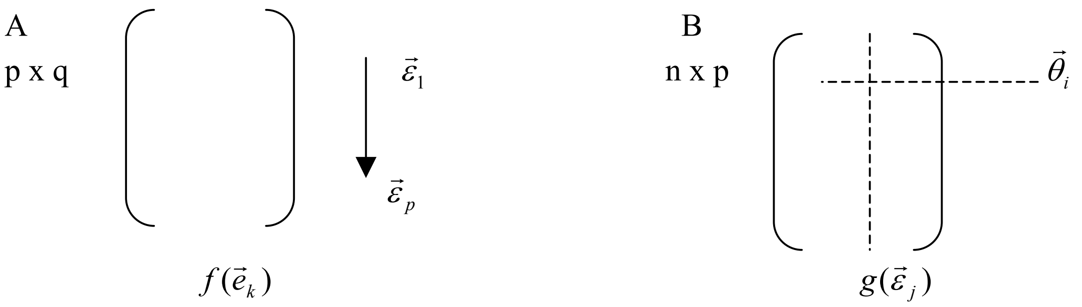
$f+g \quad E \xrightarrow{A+B} F$

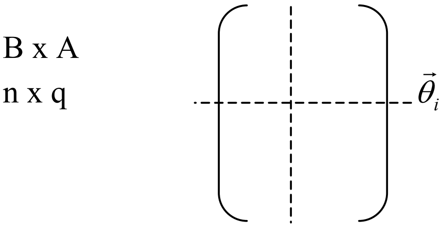
$\lambda f \quad E \xrightarrow{\lambda A} F$

si on veut composer des applications :



$\mathscr{B} = (\vec{e}_1,\dots\vec{e}_q)$ $\mathscr{B}'$ $= (\vec{\varepsilon}_1,\dots\vec{\varepsilon}_p)$ $\mathscr{B}''$ $= (\vec{\theta}_1,\dots\theta_n)$



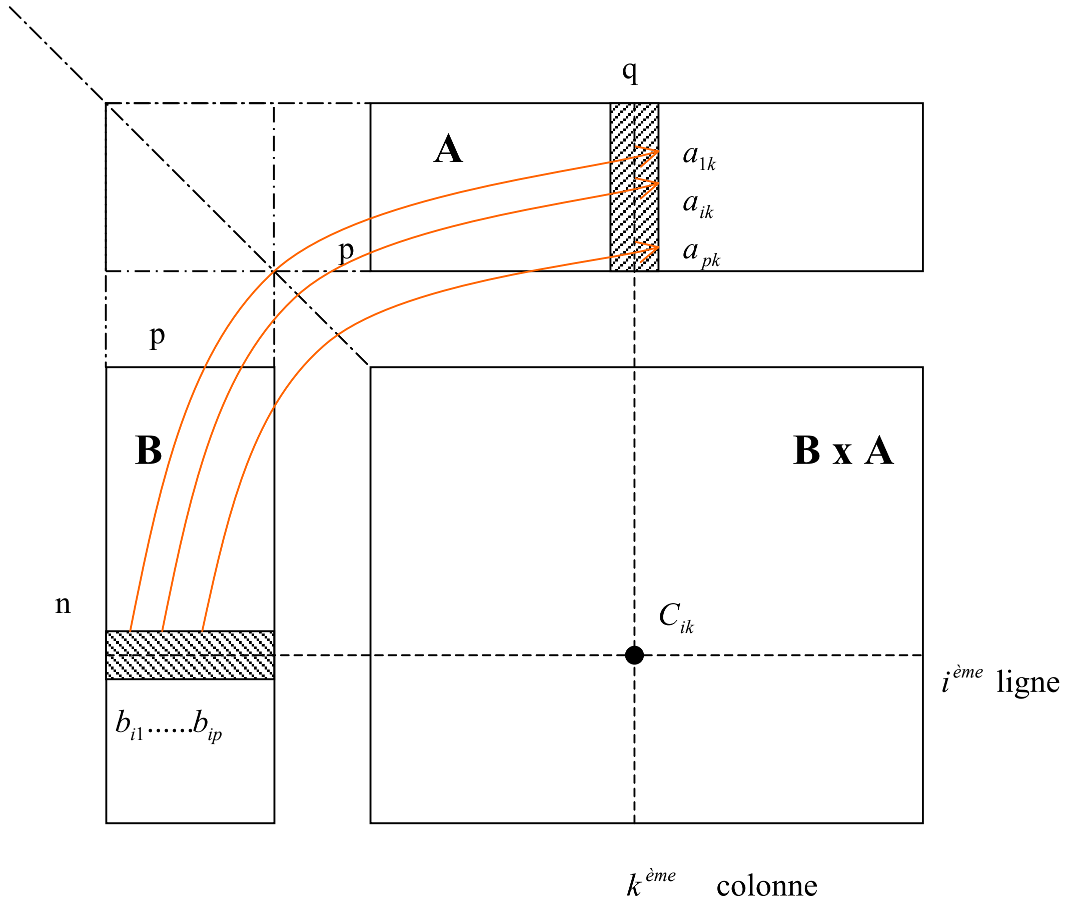


$$gof(\vec{e}_k)$$

$$= g[a_{1k} \bullet \vec{\varepsilon}_1 + ... + a_{jk}\vec{\varepsilon}_j + ... + a_{pk} \bullet \vec{\varepsilon}_p] = a_{1k} \bullet g(\vec{\varepsilon}_1) + ... + a_{jk} \bullet g(\vec{\varepsilon}_j) + ...a_{pk} \bullet g(\vec{\varepsilon}_p)$$

$$+ [a_{1k} \bullet b_{ij} + ... + a_{jk} \bullet b_{ij} + ... + a_{pk} \bullet b_{ip}]\vec{\theta}_i$$

$$\boxed{C_{ik} = b_{i1} \bullet a_{1k} + ... + b_{ij} \bullet a_{jk} + ...b_{ip} \bullet a_{pk} = \sum_{j=1}^{p} b_{ij} \bullet a_{jk}}$$



exercice :

multiplier : A $\begin{matrix} 0 & 0 \\ 1 & 1 \end{matrix}$ B $\begin{matrix} 1 & 0 \\ 1 & 0 \end{matrix}$

Vérifier $A^2 - B^2 \neq (A - B)x(A + B)$

Ex : A = $\begin{matrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{matrix}$ C = $\begin{matrix} 2 & -3 & 0 & 1 \\ 5 & -1 & 4 & 2 \\ -1 & 0 & 0 & 3 \end{matrix}$ B = $\begin{matrix} 4 & 0 & -3 \\ -1 & -2 & 3 \end{matrix}$ D = $\begin{matrix} 2 \\ -1 \\ 3 \end{matrix}$

Faire AB, BA, AD, DA, ……

Ex A= $\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$ $A^2, A^3,....A^n$ pour tout n $\in \mathbb{N}$